# Lectures on Contest Mathematics

## US1 – Invitation to Contest Algebra and Geometry

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*"Cogito ergo Sum" – "I think, therefore I am"*

René Descartes (1596-1650)

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# Chapter 1

# Algebra





## 1.1 Operations rules with exponents

1. We write $4^3 = 4 \cdot 4 \cdot 4$ or $4^3 = 4 \times 4 \times 4$. The small raised number is called an *exponent*, and $4^3$ is *a power of 4*. Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ as a power of 4.

2. Write the product $4^3 \cdot 4^5$ as a power of 4.

3. [PEA Math1 Materials] Faced with the problem of multiplying $5^6$ times $5^3$, Brook is having trouble deciding which of these four answers is correct: $5^{18}$, $5^9$, $25^{18}$, or $25^9$. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are able to formulate the *common-base principle for multiplication* of expressions $b^m \cdot b^n$.

4. [PEA Math1 Materials] Exponents are routinely encountered in scientific work, where they help investigators deal with large numbers:

   (a) The human population of Earth is roughly 6000000000, which is usually expressed in *scientific notation* as $6 \times 10^9$. The average number of hairs on a human head is $5 \times 10^5$. Use scientific notation to estimate the total number of human head hairs on Earth.

   (b) Light moves very fast — approximately $3 \times 10^8$ meters every second. At that rate, how many meters does light travel in one year, which is about $3 \times 10^7$ seconds long? This so-called *light year* is used in astronomy as a yardstick for measuring even greater distances.

5. [PEA Math1 Materials] In $7^4 \cdot 7^4 \cdot 7^4 = \left(7^4\right)^{\Delta}$ and $b^9 \cdot b^9 \cdot b^9 \cdot b^9 = \left(b^9\right)^{\nabla}$, replace the triangles by correct exponents. The expression $\left(p^5\right)^6$ means to write $p^5$ as a factor how many times? To rewrite this expression without exponents as $p \cdot p \cdot p \cdots$, how many factors would you need?

6. [PEA Math1 Materials] The diameter of an atom is so small that it would take about $10^8$ of them, arranged in a line, to span one centimeter. It is thus a plausible estimate that a