

# Lectures on Contest Mathematics

## US2 Contest Supplement for MathCounts, AMC10, and AIME

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*“Cogito ergo Sum” – “I think, therefore I am”*

René Descartes (1596-1650)

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