

## Solving Systems of Equations

- Solving a system of equations in two variables by the substitution method
- Solving a system of equations in two or three variables by linear combinations

### Example: Solve

$$\begin{array}{lll}
 \mathbf{1} \quad (1) & \begin{array}{l} 2x + y = 6 \\ 3x + 4y = 4 \end{array} & (2) \quad \begin{array}{l} 5x + 4y = 11 \\ 3x - 5y = -23 \end{array} & (3) \quad \begin{array}{l} \frac{1}{x} - \frac{3}{y} = 2 \\ \frac{6}{x} + \frac{5}{y} = -34 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \mathbf{2} \quad 2x - 4y + 7z = 24 \\
 \quad \quad 4x + 2y - 3z = 4 \\
 \quad \quad 3x + 3y - z = 4
 \end{array}$$

**3** Determine a, b, and c if  $(2, 3, -4)$  is a solution of the system

$$\begin{array}{l}
 ax + by + cz = -11 \\
 bx - cy + az = -19 \\
 ax + cy - bz = 9
 \end{array}$$

### Challenge:

**1** A box contains some green marbles and exactly four red marbles. The probability of selecting a red marble is  $x\%$ . If the number of green marbles is doubled, the probability of selecting one of the four red marbles from the box is  $(x - 15)\%$ . How many green marbles are in the box before the number of green marbles is doubled?

**2** The table to the right represents the relationship between  $x$  and  $y$  in a quadratic equation of the form  $y = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are integers. What is the value of  $a$ ?

<b>x</b>	1	2	3	4	5	6	7
<b>y</b>	-8	5	24	49	80	117	160

**3** Joe bought a pumpkin that cost 10 cents more per pound than his sister's. Together, the two pumpkins weighed 20 pounds, but Joe's pumpkin was heavier. Joe paid \$3.52, and his sister paid \$0.48. How many pounds did Joe's pumpkin weigh?

**4** The total number of degrees in the sum of the interior angles of two regular polygons is 1980. The sum of the number of diagonals in the two polygons is 34. What is the positive difference between the numbers of sides of the two polygons?

**5** Suppose  $a$ ,  $b$ ,  $c$ , and  $d$  are integers satisfying:  $a - b + c = 3$ ,  $b - c + d = 4$ ,  $c - d + a = 1$  and  $d - a + b = 6$ . What is the value of  $a + b + c + d$ ?

**6** The variables  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$  each represent exactly one of the integers 1 through 6. Given the following facts, which integer is represented by  $e$ ?

$$a + b = c; \quad b + c = d; \quad e + c = f.$$

### Exercise

**1** Groucho, Zippo, and Harpo have \$65 altogether. Groucho and Zippo together have \$1 more than Harpo has, and Groucho has \$15 more than Zippo. How many dollars does Groucho have?

**2** On Monday, 30 girls and 0 boys in the Crescent Valley Band missed practice, resulting in a 2 : 5 ratio of girls to boys. On Tuesday, 30 girls and 48 boys missed practice, resulting in a 2 : 1 ratio of girls to boys. How many students are in the band?

**3** The letters of alphabet are each assigned a random integer value, and  $H = 10$ . The value of a word comes from the sum of its letters' values. If MATH is 35 points, TEAM is 42 points and MEET is 38 points, what is the value of A?

**4** Marianna has only nickels and quarters in her piggy bank. Their combined value is \$9.15. Their combined weight is one pound. Ninety nickels weigh one pound. Eighty quarters weigh one pound. How many nickels does Marianna have in her piggy bank?

**5** A collection of 20 coins made up of only nickels, dimes and quarters has a total value of \$3.35. If the dimes were nickels, the nickels were quarters and the quarters were dimes, the collection of coins would have a total value of \$2.75. How many quarters are in the original collection?

**6** At the Word Store, each vowel sells for a different price, but all consonants are free. The word "triangle" sells for \$6, "square" sells for \$9, "pentagon" sells for \$7, "cube" sells for \$7 and "tetrahedron" sells for \$8. What is the dollar cost of the word "octahedron"?

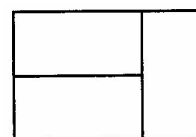
**Homework**

**1** Given that  $9a^2 - 8b^2 = 1800$  and  $a/b = 4/3$ , what is the value of the product  $ab$ ?

**2** The sum of three numbers is 105. The third is 11 less than ten times the second. Twice the first is 7 more than three times the second. Find the numbers.

**3** Given the equations  $3x + y = 17$ ,  $5y + z = 14$  and  $3x + 5z = 41$ , what is the value of the sum  $x + y + z$ ?

**4** This figure has a total area of 108 square units and is a rectangle composed of three congruent, small rectangles. Find the sum of the length and width of one of the smaller rectangles. Express your answer in simplest radical form.



**5** A data set for a class of 25 sixth graders has their ages listed as the integer values of either 10 or 11 years. The median age in the data set is 0.36 years greater than mean. How many 10-year-olds are in the class?

**6** A closed cardboard box is twice as wide and three times as long as it is deep. If the total surface area is 137.5 square feet, then how many feet are in the sum of the length, width, and depth of the box?