

Triangle

Some Theorems and Postulates:

- 1 The sum of the measures of the angles of a triangle is 180.
- 2 The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.
- 3 The sum of the measures of the exterior angles of a triangle, one angle at each vertex, is 360.
- 4 (**The Triangle Inequality**) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- 5 If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. (**SSS**)
- 6 If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. (**SAS**)
- 7 If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. (**ASA**)
- 8 If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. (**AA Similarity**)
- 9 If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are in proportion, then the triangles are similar. (**SAS Similarity**)
- 10 If the sides of two triangles are in proportion, then the two triangles are similar. (**SSS Similarity**)
- 11 If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.
- 12 If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.
- 13 (**Pythagorean Theorem**) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.
- 14 In a $45^\circ - 45^\circ - 90^\circ$ triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.
- 15 In a $30^\circ - 60^\circ - 90^\circ$ triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

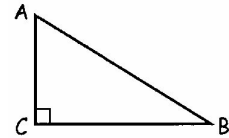
Examples:

1 (06 National/Target 4) Among all triangles with integer side length and perimeter 20 units, what is the area of the triangle with the largest area? Express your answer in simplest radical form.

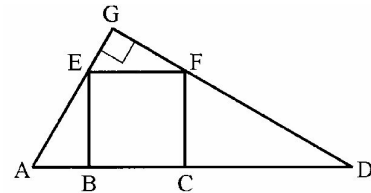
2 (04-05 Wout 8-4) A right triangle has sides measuring $2x - 1$, $3x - 13$, and $3x - 4$ units. What is the value of x ?

3 (03-04 Wup 17-10) The endpoints of segment AB are $A(0, 8)$ and $B(15, 0)$. What is the shortest distance, in units, from $P(0, 0)$ to segment AB ?

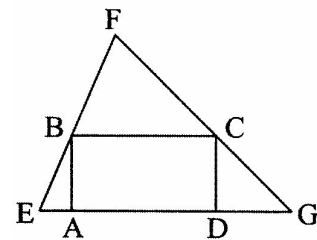
4 (03-04 Wout 8-7) Medians are drawn from point A and point B in this right triangle to divide segment BC and AC in half, respectively. The lengths of the medians are 6 and $2\sqrt{11}$ units, respectively. How many units are in the length of segment AB ?



5 (05 State/Sprint 29) Square $BCFE$ is inscribed in right triangle AGD , as shown to the right. If $AB = 28$ units and $CD = 58$ units, what is the area of square $BCFE$?

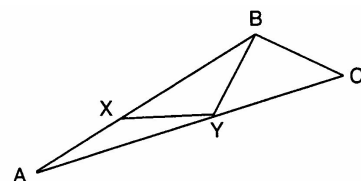


6 (07 National/Team 8) Rectangle $ABCD$ is inscribed in triangle EFG such that side AD of the rectangle is on side EG of the triangle, as shown. The triangle's altitude from F to side EG is 7 inches, and $EG = 10$ inches. The length of segment AB is equal to half the length of segment AD . What is the area of rectangle $ABCD$? Express your answer as a common fraction.

**Exercises:**

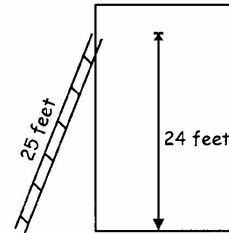
1 (92-93 Chapter/Team 5) How many distinct triangles have a perimeter of 12 units and integer length sides?

2 (92-93 National/Target 7) In triangle ABC , $AX = XY = YB = BC$ and the measure of $\angle ABC$ is 120° . What is the degree measure of $\angle BAC$?



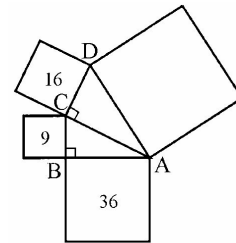
3 ((02-03 Wup 13-6) One leg of a right triangle is two meters longer than twice the length of the other leg. The hypotenuse is eight meters longer than the shorter of the two legs. What is the perimeter of the triangle, in meters?

4 (02-03 Wup 12-6) A 25-foot ladder reaches 24 feet up the side of building. Then the top of the ladder slides down 4 feet. How many additional feet does the bottom of the ladder slide out from the base of the building?

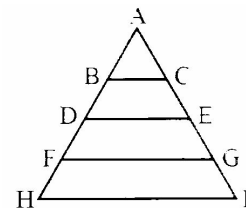


5 (03-04 Wout 9-8) There are two non-congruent triangles. Each triangle has sides of length 10 cm and 12 cm, and the measure of the angle opposite the 10-centimeter sides is 45 degree in each triangle. In square centimeters, what is the sum of the areas of these two triangles?

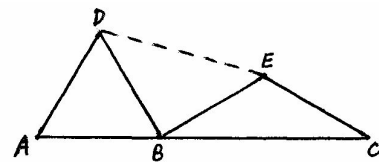
6 (04 State/Target 6) Two right triangles ABC and ACD, are joined as shown. Squares are drawn on four of the sides. The areas of three of the squares are 9, 16, and 36 square units. What is the number of square units in the area of the fourth square?



7 (04 State/Sprint 27) Triangle AHI is equilateral. We know BC, DE, and FG are all parallel to HI and $AB = BD = DF = FH$. What is the ratio of the area of trapezoid FGIH to the area of triangle AHI? Express your answer as a common fraction.



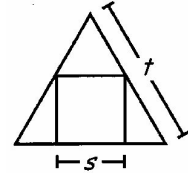
8 (05 State/Team 6) Coplanar points A, B, C, D, and E are arranged such that A, B, and C are collinear with B between A and C, triangle ABD is equilateral, triangle BEC is isosceles with congruent legs BE and EC, and points D and E are on the same side of line AC. The measure of $\angle EBC$ is 30 degrees. The areas of triangle ABD and triangle BEC are equal. What is the number of degrees in the measure of $\angle BDE$?



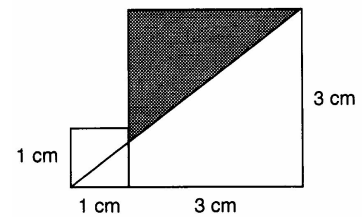
Homework:

1 (92-93 State/Sprint 26) How many distinct isosceles triangles having sides of integral length and perimeter 113 are possible?

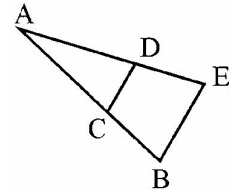
2 (03-04 Wout 9-6) A square with side length s is inscribed as shown in an equilateral triangle with side length t . What is the ratio $t : s$? Express your answer as a decimal to the nearest thousandth.



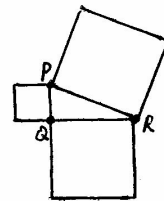
3 (92-93 National/Target 1) What is the area in square centimeters of the shaded region in the figure? Express your answer as a decimal.



4 (04 Chapter/team 4) Isosceles triangle ABE of area 100 square inches is cut by CD into an isosceles trapezoid and a smaller isosceles triangle. The area of the trapezoid is 75 square inches. If the altitude of triangle ABE from A is 20 inches, what is the number of inches in the length of CD?



5 (03 State/ Sprint 20) Angle PQR is a right angle. The three quadrilaterals shown are squares. The sum of the areas of the three squares is 338 square centimeters. What is the number of square centimeters in the area of the largest square?



6 (01-02 Wup 5-2) What is the number of inches in the perimeter of quadrilateral ABCD?

